Closing Tue: TN 4 Closing Thu: TN 5

Entry Tasks (Sigma Notation Practice)

1. Differentiate and integrate:

$$f(x) = \sum_{k=2}^{5} \frac{(-1)^k}{k^3} x^k = \frac{1}{8}x^2 - \frac{1}{27}x^3 + \frac{1}{64}x^4 - \frac{1}{125}x^5$$

2. Combine

$$5\sum_{k=2}^{4} k^2 x^k - 6\sum_{k=2}^{4} \frac{1}{k!} x^k$$

TN 5: Using Taylor Series

Here are the 6 series you can quote:

$$e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k \ , \qquad \text{for all x}$$

$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} \ , \text{for all x}$$

$$\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k} \ , \qquad \text{for all x}$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \ , \qquad \text{for } -1 < \text{x} < 1$$

$$-\ln(1-x) = \sum_{k=0}^{\infty} \frac{1}{k+1} x^{k+1} \ , -1 < \text{x} < 1$$

$$\arctan(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1} , -1 < \text{x} < 1$$

Tools for using Taylor Series

1. Substitute (replace x)
2. Integrate

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$$

3. Differentiate

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

4. Combine

$$\sum_{k=0}^{\infty} kx^k - 3\sum_{k=0}^{\infty} \frac{1}{k!} x^k = \sum_{k=0}^{\infty} \left(k - \frac{3}{k!}\right) x^k$$

Substitution Questions: Find the Taylor series based at 0, find the first three nonzero terms and give the interval of convergence.

(a) $f(x) = 3e^{2x}$

(b) $g(x) = \frac{5}{1-4x}$

(c) $h(x) = \frac{3}{2x+1}$

Combining: Find the Taylor series based at 0, find the first three nonzero terms and give the interval of convergence

(a) $y = 7 + 3x^5 e^{2x}$

(b) $y = \frac{5}{1-4x} - \frac{3}{2x+1}$

(c) $y = \cos^2(x)$ (Hint: Half-angle)

Integrating Applications

(a) Give the first three nonzero terms of the Taylor Series for

$$\int_0^x 7 + 3t^5 e^{2t} dt$$

(b) Find a Taylor series for:

$$A(x) = \int_0^x \frac{\sin(t)}{t}\, dt$$